Can experimental tests of Bell inequalities performed with pseudoscalar mesons be definitive?

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Abstract

We discuss if experimental tests of Bell inequalities performed with pseudoscalar mesons (K or B) can be definitive. Our conclusion is that this is not the case, for the efficiency loophole cannot be eliminated.

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The idea that Quantum Mechanics (QM) could be an incomplete theory, representing a statistical approximation of a complete deterministic theory (where observable values are

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fixed by some hidden variable) appeared already in 1935 thank to the celebrate Einstein-Podolsky-Rosen paper [1].

A fundamental progress in discussing possible extensions of QM was the discovery of Bell [2] that any realistic Local Hidden Variable LHV theory must satisfy certain inequalities which can be violated in QM leading in principle to a possible experimental test of the validity of QM as compared to LHV.

Since then, many interesting experiments (in practice all based on entangled photon pairs) have been devoted to a test of Bell inequalities [3–7], leading to a substantial agreement with standard quantum mechanics (SQM) and strongly disfavouring LHV theories, but, so far, no experiment has yet been able to exclude definitively such theories. In fact, so far, one has always been forced to introduce a further additional hypothesis [9], due to the low total detection efficiency, stating that the observed sample of particle pairs is a faithful subsample of the whole. This problem is known as *detection or efficiency loophole*. The search for new experimental configurations able to overcome the detection loophole is of course of the greatest interest.

In the 90's big progresses in this direction have been obtained by using parametric down conversion (PDC) processes for generating entangled photon pairs with high angular correlation. The generation of entangled states by parametric down conversion (PDC) has replaced other techniques, such as the radiative decay of excited atomic states, as it was in the celebrated experiment of A. Aspect et al. [4], for it overcomes some former limitations. Many interesting experiments have been realised using such a technique. The first experiments had, by construction, a limited total efficiency [5,6,8] and were far from eliminating the detection loophole [9]. More recently, an experiment, based on Type II PDC [7], has obtained a much higher total efficiency than the previous ones (around 0.3), which is, however, still far from the required value of 0.81. Also, some recent experiments studying equalities among correlations functions rather than Bell inequalities [11] are far from solving these problems [12]. A large interest remains therefore for new experiments increasing total quantum efficiency in order to reduce and finally overcome the efficiency loophole.

Some years ago, a very important theoretical step in this direction was performed recognising that, while for maximally entangled pairs a total efficiency larger than to 0.81 is required to obtain an efficiency-loophole free experiment, for non maximally entangled pairs this limit is reduced to 0.67 [10] (in the case of no background). An experiment addressed to test Bell inequalities using non-maximally entangled photon pairs has been recently realised [13]. Work is in progress for obtaining an efficiency above 0.67 with this kind of set-ups.

Even if relevant progresses toward the elimination of the detection loophole have been obtained using entangled photon pairs, nevertheless the total efficiency is strongly dominated by the quantum efficiency of photodetectors. Nowadays efficiencies for commercial photodetectors are around 70 %. Prototypes already reach much higher efficiencies [14], but at the prize of high background which also limits the possibility of a loophole free test [17]. Thus, in summary, the use of entangled photon pairs has led to very important tests of Bell inequalities, but at the moment does not allow to eliminate the detection loophole.

On the other hand, a recent experiment [15] performed using Be ions has reached very high efficiencies (around 98 %), but in this case the two subsystems (the two ions) are not really separated systems and the test cannot be considered a real implementation of a detection loophole free test of Bell inequalities [16], even if constitutes a relevant progress in this sense.

Even if little doubts remain on the validity of the standard quantum mechanics, considering the fundamental importance of the question, the search for other experimental schemes for a definitive test of Bell inequalities is therefore of the largest interest.

In the last years many papers have been devoted to study the possibility of realising such a test by the use of pseudoscalar meson pairs as $K\bar{K}$ or $B\bar{B}$. If the pair is produced by the decay of a particle at rest in the laboratory frame (as the ϕ at Daphne), the two particles can be easily separated to a relatively large distance allowing an easy space-like separation of the two subsystems and permitting an easy elimination of the space-like loophole, i.e. realising two completely space-like separated measurements on the two subsystems (where the space-like separation must include the setting of the experimental apparata too). A very

low noise is expected as well.

The idea is to use entangled states of the form:

$$|\Psi\rangle = \frac{|K_0\rangle|\bar{K}_0\rangle - |\bar{K}_0\rangle|K_0\rangle}{\sqrt{2}} = \\ = \frac{|K_L\rangle|K_S\rangle - |K_S\rangle|K_L\rangle}{\sqrt{2}} \tag{1}$$

Claims that these experiments could allow the elimination of the detection loophole for the high efficiency of particles detectors, have also been made. In this letter we study critically this statement.

The main caveat derives from the fact that in any experimental test proposed up to now one must tag the P or $\bar{P}$ trough its decay. This requires the selection of $\Delta S = \Delta Q$ semileptonic decays, which represent only a fraction of the total possible decays of the meson, e.g $BR(K_S^0 \to \pi^+ e^- \nu_e) = (3.6 \pm 0.7)10^{-4}$,$BR(K_L^0 \to \pi^+ e^- \nu_e) = 0.1939 \pm 0.0014$, $BR(K_L^0 \to \pi^+ \mu^- \nu_\mu) = 0.1359 \pm 0.0013$, $BR(B^0 \to l^+ \nu_l X) = 0.105 \pm 0.008$ [22]). Furthermore, experimental cuts on the energies of the decay products will inevitably reduce further this fraction and part of the pairs could be lost by decays occurring before the region of observation. Finally, most of these proposals involve the regeneration phenomenon, which introduces further strong losses. Thus, one is led to subselect a fraction of the total events. As one cannot exclude a priori hidden variables related to the decay properties of the meson, one cannot exclude the sample to be biased and thus the detection loophole pops out again. This is in analogy to the photon experiments, where the detection loophole derives by the fact that one cannot exclude losses related to the values of hidden variables which determine if the photon passes or not a polarisation (or another) selection. Namely, in a local realistic model its properties are completely specified by the hidden variables. Also decays, in a deterministic model, can happen according to the values of the hidden variables (both in a deterministic or in a probabilistic way). Thus, states with different hidden variables can decay in different channels, with the condition that the branching ratios *averaged* on the hidden variables distribution reproduce the quantum mechanics predictions.

For the experiments based on Bell inequalities measurements [18], the limits discussed before for the total efficiency remain valid. As the total branching ratio in $\Delta S = \Delta Q$ semileptonic decays is much smaller than 0.81 (the eventual use of non-maximally entangled states, lowering the efficiency threshold to 0.67, does not change the situation), this inevitably implies that a loophole free test of Bell inequalities cannot be performed in this case (even neglecting other problems [19]). It must be noticed that this problem does not appear in Ref. [23], however other additional hypotheses are needed (see Eq. 15 and discussion after Eq. 18 of [23]), and thus this proposal does not allow a general test of HVT as well.

It must also be noticed that the only observation of interference between the two term of the entangled wave function, Eq. 1, as in Ref [24], does not exclude general HVT, for this feature can be reproduced in an general class of local realistic theories.

Let us then consider other proposals not based on a Bell inequalities measurement. Two proposals of this kind have been recently advanced by F. Selleri and others concerning a $K\bar{K}$ [20] or a $B\bar{B}$ [21] system respectively.

Let us begin analysing the $K\bar{K}$ case (the $B\bar{B}$ one follows with small modifications.)

In the model of Ref. [20], the $K\bar{K}$ pair is local-realistically described by means of two hidden variables, one (λ_1) determining a well defined CP value, the other (λ_2) a well defined strangeness S value for the K (and related to this for the $\bar{K}$). This second variable cannot be a time independent property, but is subject to sudden jumps. If locality must be preserved the time of this jump must already be fixed ab initio by a hidden variable (which represents the real second hidden variable of the model) and the two subsystems must not influence each other while they are flying apart, namely λ_2 is not the true hidden variable, but a parameter driven by the true hidden variable (see appendix of Ref. [20]).

Let us denote by K_1 the state with CP=1, S=1, K_2 the state with CP=1, S=-1, K_3 the state with CP=-1, S=1 and K_4 the state with CP=-1, S=-1.

The initial state can be, with probability 1/4, in anyone of the states $CP = \pm 1$, $S = \pm 1$. Each of these pairs give, in the local-realistic model (LRM), a certain probability of observing

a $\bar{K}_0\bar{K}_0$ pair at proper times t_a and t_b ($\neq t_a$) of the two particles. These probabilities are (in a somehow simplified form, see eq. 62-70 of Ref. [20]):

$$
\begin{aligned}
P_1[t_a, t_b] &= [E_S(t_a)Q_-(t_a) - \rho(t_a)] \cdot E_L(t_a)p_{43}(t_b|t_a) \\
P_2[t_a, t_b] &= [E_S(t_a)Q_+(t_a) + \rho(t_a)] \cdot E_L(t_a)p_{43}(t_b|t_a) \\
P_3[t_a, t_b] &= [E_L(t_a)Q_-(t_a) + \rho(t_a)] \cdot E_S(t_a)p_{21}(t_b|t_a) \\
P_4[t_a, t_b] &= [E_L(t_a)Q_+(t_a) - \rho(t_a)] \cdot E_S(t_a)p_{21}(t_b|t_a) \quad (2)
\end{aligned}
$$

corresponding to an initial state with K_1 on the left and K_4 on the right, K_2 on the left and K_3 on the right, K_3 on the left and K_2 on the right and K_4 on the left and K_1 on the right respectively.

In Eq. 2, we have introduced $E_S(t) = exp(-\gamma_S t)$ and $E_L(t) = exp(-\gamma_L t)$, where $\gamma_S = (1.1192 \pm 0.0010)10^{10}s^{-1}$ and $\gamma_L = (1.934 \pm 0.015)10^7 s^{-1}$ denote the decay rate of K_S and K_L [22]. $Q_\pm = \frac{1}{2}\left[1 \pm \frac{2\sqrt{E_L E_S}}{E_L + E_S}\cos(\Delta m t)\right]$, where $\Delta m = M_L - M_S = (0.5300 \pm 0.0012)10^{10}s^{-1}$. Furthermore, we have defined $p_{21}(t_b|t_a) = E_S^{-1}(t_a)[p_{21}(t_b|0) - p_{21}(t_a|0) \cdot E_S(t_b - t_a)]$ and $p_{43}(t_b|t_a) = E_L^{-1}(t_a)[p_{43}(t_b|0) - p_{43}(t_a|0) \cdot E_L(t_b - t_a)]$ where $p_{21}(t|0) = E_S(t)Q_-(t) - \rho(t)$ and $p_{43}(t|0) = E_L(t)Q_-(t) + \rho(t)$. Finally, $\rho(t)$ is a function not perfectly determined in the model (see discussion in Ref. [20]), but which is limited by

$$
\begin{aligned}
-E_S Q_+ \leq \rho \leq E_S Q_- \\
-E_L Q_- \leq \rho \leq E_L Q_+
\end{aligned}
\qquad (3)
$$

If the total efficiency is 1, the LRM probability of observing a $\bar{K}_0\bar{K}_0$ pair is given by the sum of the four probabilities of Eq. 2 multiplied for 1/4. It is rather different from the quantum mechanical prediction and thus represents a good test of the LRM (see fig. 1, where $P[\bar{K}_0(t_a), \bar{K}_0(2t_a)]$ is reported in analogy to Table 1 of Ref. [20]). Nevertheless, when the total efficiency is lower than 1, the different probabilities can contribute in different way as the hidden variables, which determines the passing or not the test, could also be related to the decay properties of the meson pair. As discussed previously, the specific property of

the meson is not being or not a $\bar{K}_0$ at a certain proper time, but the hidden variables values characterise it completely, and thus, in principle, even its decay properties. If this is the case, different coefficients a_i can multiply the four probabilities. One has therefore:

$$P[\bar{K}_0(t_a), \bar{K}_0(t_b)] = 1/4 \cdot [a_1 P_1[t_a, t_b] + a_2 P_2[t_a, t_b] + a_3 P_3[t_a, t_b] + a_4 P_4[t_a, t_b]] \tag{4}$$

The freedom of the choice of this parameters allow to reproduce the quantum mechanical prediction. In figure 2 we report the case corresponding to a total efficiency of 0.3 (other values can be obtained by scaling): the lowest limit curve of local realism can easily reproduce or be lower than the quantum mechanics prediction when different weights multiply the four probabilities, due to different branching ratios for the 4 cases. As an example, the curve corresponding to LRM with weights $a_1 = 1$,$a_2 = 0.07$,$a_3 = 0.03$ and $a_4 = 0.1$ is shown. For the sake of simplicity, in this example the values of a_i are chosen such that their sum is the total efficiency, but the value of the a_i are substantially very little constrained as they can depend on the time: only the total fraction of observed decays should be reproduced. Furthermore, the result is obtained with $\rho = 0$. Thanks to the large arbitraryness of ρ, other choices would even allow to reproduce easier the quantum mechanics result (as we have calculated for a lot of different choices of ρ). It must be emphasised that the LRM curve in fig.2 is not a fit to the quantum mechanics curve, but only a simple choice showing the effect we are discussing. Considering the large arbitrary of a general HVT, our purpose is only to show a counterexample, which proves that observation of the curve predicted by SQM cannot exclude *every* HVT. Of course, as all Bell inequalities tests performed up to now, a result in agreement with SQM will further reduce the space for the existence of a realistic HVT, even if unable to obtain the general result of eliminating the possible existence of HVT.

Let us notice that situation 1 and 4, 2 and 3 are symmetric under the exchange of left and right, however the decay probabilities need not to be the same. Furthermore, let us emphasise again that, in principle, inside this model the coefficients could even be function of the time $a_i(t)$, like the value for the hidden variable λ_2. This property of course makes easier to reproduce the quantum mechanics prediction. Thus, when only a subsample is

selected (semileptonic $\Delta S = \Delta Q$ decays must be observed for tagging a $\bar{K}_0 \bar{K}_0$ state and cuts must be introduced) the result of this analysis shows that detection loophole appears also in this case. Of course we are not discussing how the local realistic model should be, or if its complexity makes it unpalatable, we are simply investigating if every local realistic model could be excluded without any doubt by such experiments and we conclude that one is focused to introduce the additional hypothesis that the observed sample is unbiased concerning the hidden variables values. Even if for some specific function $\rho(t)$, it could not be possible, for time independent decay properties, to obtain a perfect agreement between LRM and SQM, for the function $\rho(t)$ is not fixed for a general LRM, this does not modify our conclusions.

Exactly the same considerations apply for what concerns the $B_0 \bar{B}_0$ case. A first set of papers [23] consider the possibility to test SQM measuring the term deriving by interference between the two terms in the entangled wave function of Eq. 1. However, this effect is also reproduced in any reasonable HVT [21] and thus cannot be considered a general test of local realism, but can only allow to eliminate some specific class of hidden variable theories. As the model of Ref. [21] is, with small changes due to the same decay time for both the CP eigenstates, equivalent to the one we have discussed in the previous paragraph, the same conclusions hold.

In conclusion, even if tests of local realism using pseudoscalar mesons represent an interesting new way of investigating quantum non-locality in a new sector, it still appears to us that none of the proposed schemes permits a conclusive test of local realism, for the impossibility of eliminating the detection loophole.

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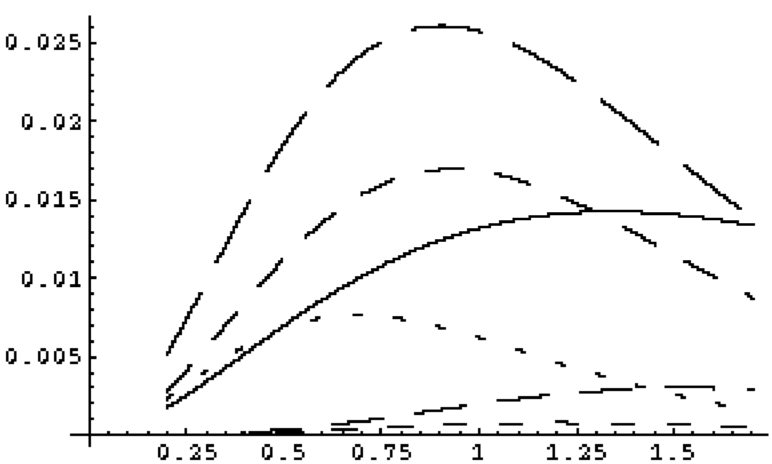
Figures Captions

Fig.1 The SQM and the minimal LRM predictions for $P[\bar{K}_0(t_a), \bar{K}_0(2t_a)]$ ($\rho = 0$). The minimal LRM is largely above the SQM prediction. For the sake of completeness we report the four probabilities P_i in function of the proper time t_a ($t_b = 2t_a$) as well. The dashing of the curves diminishes in this order.

Fig.2 The SQM (dashed) and the minimal LRM, $\rho = 0$, (thick) predictions for $P[\bar{K}_0(t_a), \bar{K}_0(2t_a)]$ keeping into account a total detection efficiency of 0.3. With the choice $a_1 = 1, a_2 = 0.07, a_3 = 0.03, a_4 = 0.1$ the two curves substantially coincides. For other

choices of the parameters the lower bound of LRM can be easily taken largely under SQM prediction.



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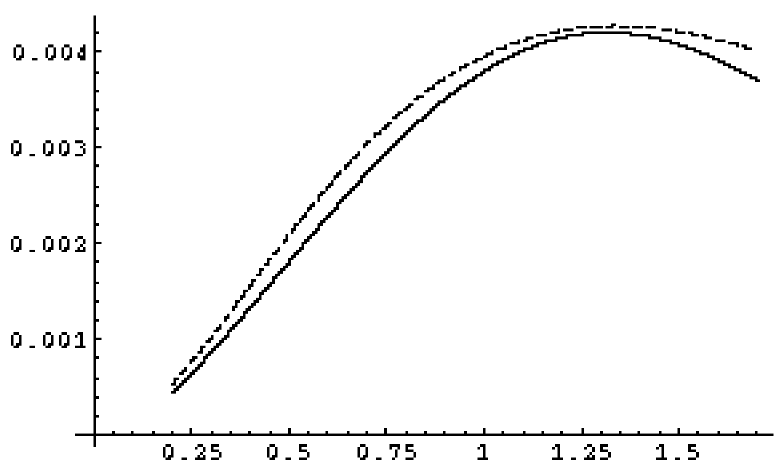
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